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NOTE

Trajectories of Trapped Particles in the Field of a Plasma Wave Excited by a Stimulated Raman Scattering

Trajectories of particles trapped and accelerated in the field of plasma waves generated by stimulated Raman scattering are calculated self-consistently by a leapfrog scheme using the electromagnetic field computed in a Vlasov simulation. The trajectories of the particles trapped in the field of the plasma wave agree with the contour plots of the distribution function calculated by the Vlasov code. These results provide a powerful method to study the trapping and escaping of particles in the field of the plasma wave. © 1993 Academic Press, Inc.

We study the problems of beatwave acceleration and current drive using Vlasov simulation [1, 2]. These codes provide a powerful tool to represent the low density regions of phase-space, which are very coarsely delineated in particle codes. Specially, Ref. [1] presents a detailed representation of the formation of vortices in the field of the plasma wave generated in a stimulated Raman scattering. The calculation of the separatrix from the Hamiltonian obtained from the self-consistent field computed with the solution of the Vlasov equation showed a very good agreement with the formation of the vortex structure [1]. The question then arises of whether one can follow the trajectories of individual particles trapped and accelerated by the plasma wave and compare these trajectories with the contour plots obtained from the Vlasov code simulation during the formation of the vortices in the field of the same plasma wave. The present work shows results obtained by integrating particle trajectories, using a leapfrog scheme, in the self-consistent field obtained from the solution of the Vlasov–Maxwell set of equations. The trajectories of the particles trapped in the field of the plasma wave conform well with the contour plots of the distribution function calculated by the Vlasov code. These results provide a powerful method to study the trapping and the escape of particles in the field of the plasma wave, where the trajectories of the particles and the fields are free from the usual statistical fluctuations associated with particles in cell codes.

The pertinent equations have been previously presented [1, 3]. We write these equations here again in order to fix the notation for the particles integration. The Vlasov equation for the electron plasma distribution function $f(x, p_x, t)$ is given by

$$\frac{\partial f}{\partial t}+\frac{p_x}{m\gamma}\frac{\partial f}{\partial x}-e(E_x+u_yB_z)\frac{\partial f}{\partial p_x}=0 \tag{1}$$

with the Lorentz factor $\gamma=(1+p_x^2/m^2c^2)^{1/2}$, such that $p_x=m\gamma u_x$. In the perpendicular y direction, we assume only a fluid monokinetic description. The terms $u_x(\partial u_y/\partial x)$ and u_xB_z simplify exactly [2], so that the relevant macroscopic equation for u_y reduces to

$$\frac{\partial u_y}{\partial t}=-\frac{e}{m}E_y. \tag{2}$$

The ions are immobile. The transverse electromagnetic fields obey Maxwell's equation as discussed in Ref. [2]. The trajectories of the particles are obtained by integrating the equations of the characteristics from Eq. (1)

$$\frac{dx}{dt}=u_x \tag{3}$$

$$\frac{dp_x}{dt}=F(x)=-e(E_x+u_yB_z), \tag{4}$$

together with Eq. (2). The force $F(x)$ is directly computed by the Vlasov code [3]. These equations are then integrated using a leapfrog scheme,

$$x^{n+1}-x^n=u_x^{n+1/2}\,\Delta t \tag{5}$$

and

$$p_x^{n+1/2}-p_x^{n-1/2}=\Delta t\,F(x^n), \tag{6}$$

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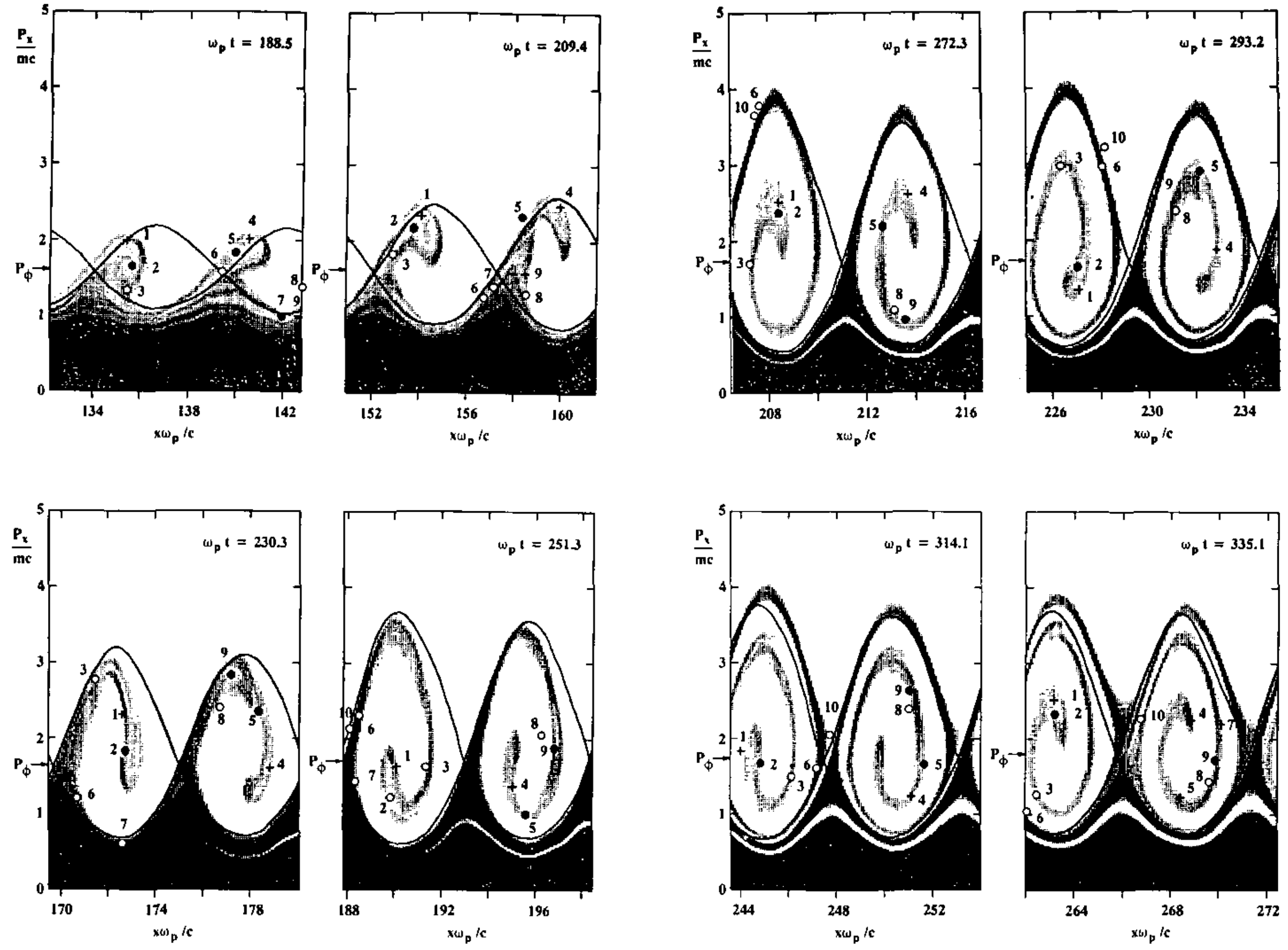

FIG. 1. Phase-space representation of the trapping region of the distribution function at different times, in a frame moving at the phase velocity of the wave. The different particles whose positions and velocities are calculated are numbered from 1 to 12.

where $F(x^n)$ is calculated by a linear interpolation scheme at time $t_n = n\,\Delta t$ using the data computed by the Vlasov code at grid points. $\Delta t = 0.0818\omega_p^{-1}$ as in Ref. [1].

Figure 1 shows the phase-space plane at different times in a frame moving with a phase velocity of the wave obtained from the solution of the Vlasov equation with the same parameters as in Ref. [1]. These figures have been presented in Ref. [1]. For reference, the electric field is given in Fig. 2 at different times, and the position of the window which is viewed in Fig. 1 is indicated by an arrow in Fig. 2. Figure 1 contains also the position of different particles (numbered from 1 to 12) whose positions and velocities are calculated using the leapfrog scheme presented in the previous section. The trajectories of the particles follow closely the contours of the distribution function, even when the vortices formed by the trapped particules have spiraled several times. For $\omega_p t$ of 356.0 and higher, Fig. 1 shows also the distribution functions associated with the phase-space at the top of the figures.

One aspect of interest is the behavior at marginal trapping. Here, for example, two particles, Nos. 6 and 10, barely not trapped at $\omega_p t = 209.4$ or $\omega_p t = 272.3$, are trapped between $\omega_p t$ values of 230.3 and 251.3. However, their fates after that are quite different. The wave amplitude is dropping, but at $\omega_p t$ value of 314.1 particle 6 remains trapped inside the trapping phase space vortex, while its neighbouring particle 10 becomes involved in the fold developing at the X-point and escapes to higher energies, now traveling on top of the vortices. Another example of interesting behavior is that of particles deeply trapped near the inside tip, where particles 1 and 2 seem to be alternating position each time the trapping spiral vortex adds another half turn,

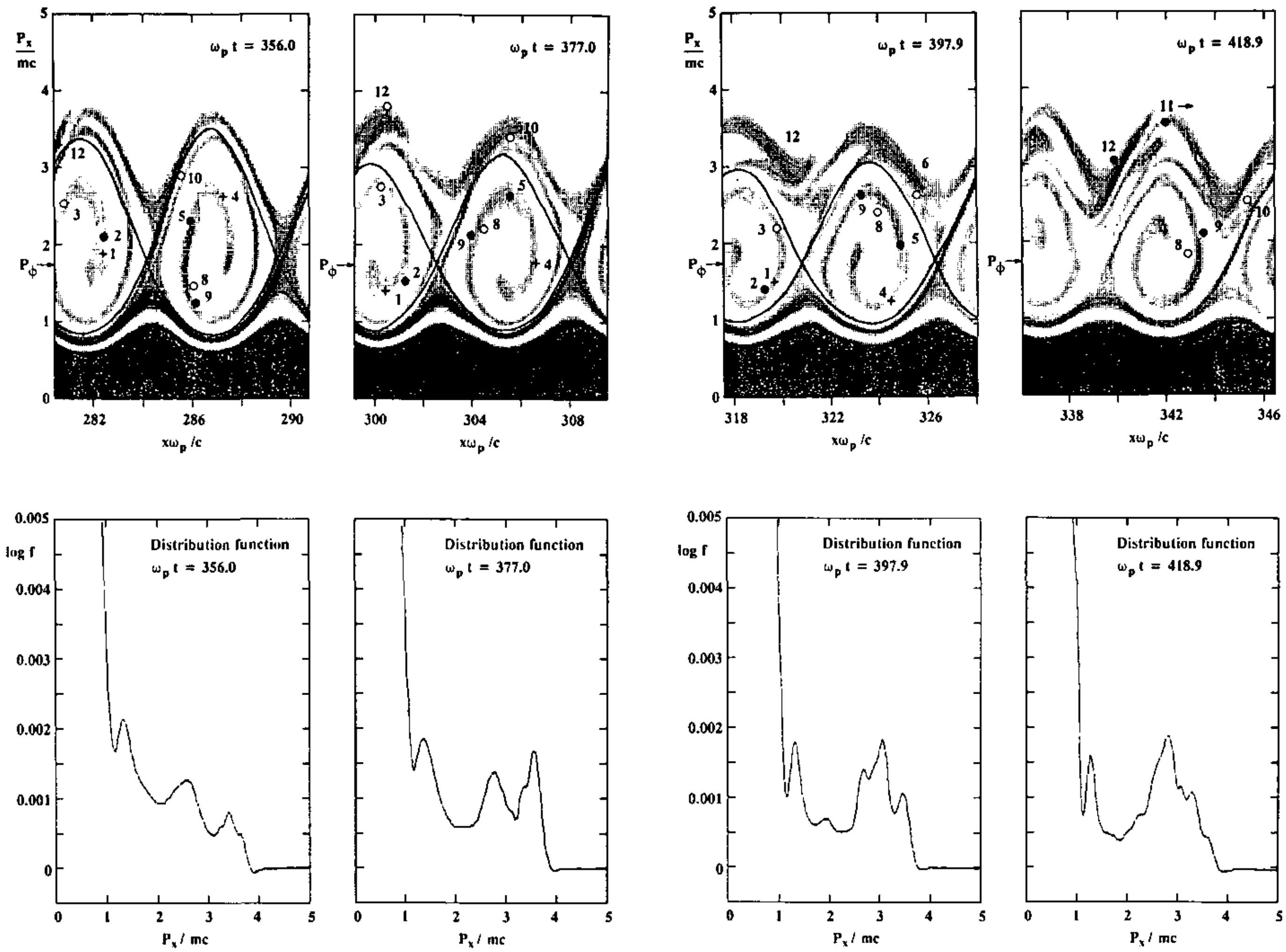

FIG. 1—*Continued*

indicating some internal rotation not evident in the phase space structure itself.

Contour plots of the distribution function calculated by the integration of the Vlasov equation, have shown the formation of spiral structure and vortices in the phase-space region corresponding to the phase velocity of the plasma wave excited by a stimulated Raman scattering [1]. The calculation of the separatrix from the Hamiltonian obtained from the self-consistent field computed with the solution of the Vlasov equation shows a very good agreement with the formation of the vortex structure [1], an indication of the good performance of the numerical scheme used to integrate the Vlasov equation. In the present work, we have followed the trajectories of particles trapped and accelerated by the plasma wave, and we compare these trajectories with the contour plots obtained from the Vlasov code simulation during the formation of the spiraling vortices in the field of the same plasma wave. The very good agreement obtained is an additional indication of the very good performance of the numerical code. This agreement (for the case with small time-step $\Delta t = 0.0818\omega_p^{-1}$ that we are studying) indicates also that the linear interpolation scheme for the calculation of the trajectories of these "marker" particles is performing nicely and is accurate enough. What effect, if any, would a higher interpolation scheme (quadratic or cubic spline) have on the behavior of barely trapped or barely passing particles (such as 6 and 10 in the figures) is a point which requires further investigation. We expect, however, that for a large number of particles, this effect to be quantitatively minimal. Only those few "marker" particles with trajectories very close to the separatrix will probably be affected. Note also that the separatrix is not exactly constant, but oscil-

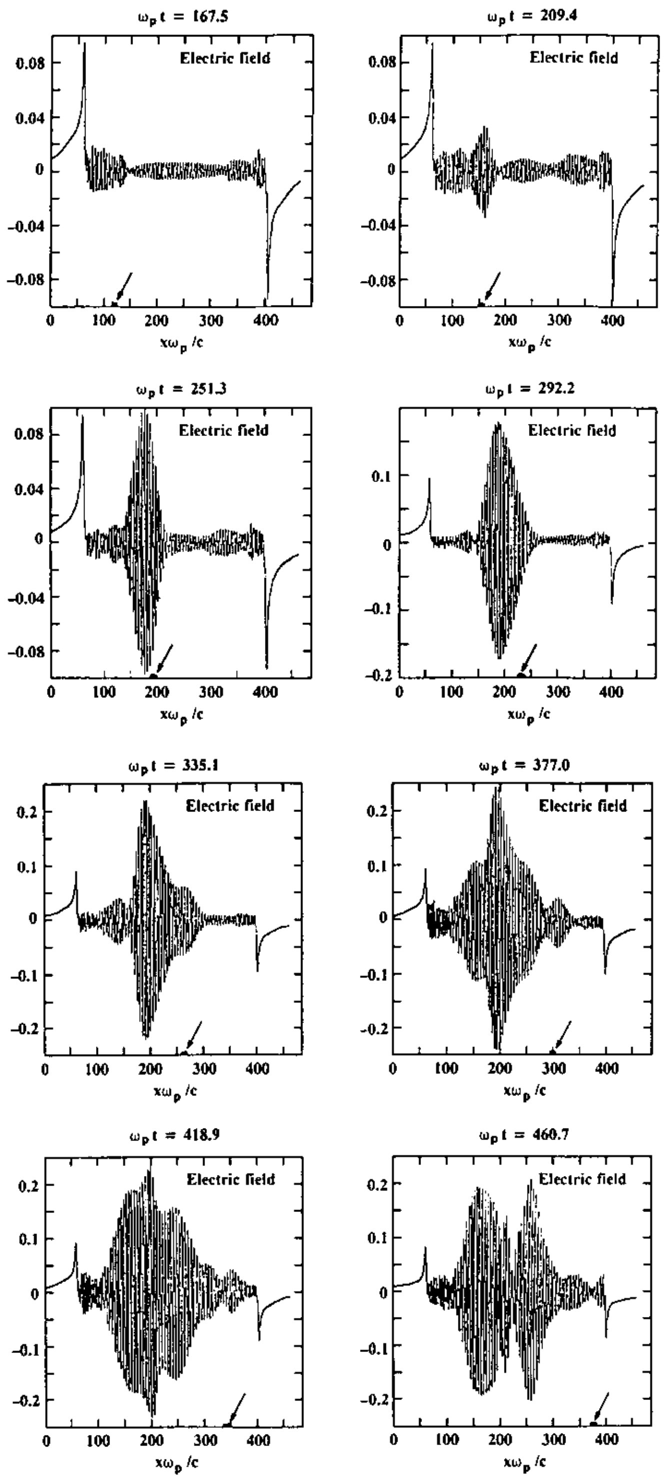

FIG. 2. Electric field at the different times selected in Fig. 1. The position of the window which is viewed in Fig. 1 is indicated by an arrow.

lating and continuously sweeping the phase-space region in its close neighborhood. It is beyond the scope of this presentation to discuss in more detail this separatrix crossing, detrapping, and folding in phase-space, but we note the relevance of the work by Cary, Escande, and Tennyson (Ref. [4] and references cited therein). The present work, however, indicates clearly that the combination of "marker" particles with Vlasov solution is a powerful tool to show some insight into trapping.

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